COMPUTER SOLUTION OF THE PROBLEM OF THE DEPLETION OF A STRATUM WITH A GASIFIED LIQUID IN THE CASE OF THE WORK OF A ROUND BATTERY OF BOREHOLES

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The article discusses the problem of the depletion of a round stratum with a gasified liquid, in the case of the work of a round battery of boreholes with constant end-face pressures. The nonlinear differential equations describing the filtration process are replaced by finite-difference equations, which are solved numerically using a computer program. The condition for the convergence of the finite-difference scheme used is given. The proposed algorithm is used to solve a concrete example. By making analogous calculations for different variants of a given number of boreholes, of the end-face pressures, of the radius of the battery, valuable information can be obtained with respect to their optimal values, the time of the start of secondary methods of exploitation, the volume of gas dissolved in the petroleum, as well as the output of petroleum, which is easily calculated from the known values of the pressure and the petroleum saturation at any given moment of time.

Articles [1-3] discuss the problem of the depletion of a stratum with a gasified liquid for the case of the work of a gallery, as well as of a straight-line battery of boreholes. Article [4] gives a calculation of the mean working indices in the case of the work of a round battery.

The picture of the distribution of the pressure and the petroleum saturation in a stratum in the case of the work of a round battery of boreholes is of great theoretical and practical importance. In the present article the problem is solved for the depletion of a round stratum with a gasified liquid, using computer programming.

It is assumed that it is desired to exploit a round stratum of radius r_2 with an impermeable external boundary. The original values of the stratum pressure and the petroleum saturation are equal to p_0 and ρ_0. On a circle of radius r_1, concentric with the external boundary, there is arranged a battery, perfect with respect to its degree of opening-up, and consisting of n equally spaced boreholes. At the moment of time $t=0$ the end-face pressures of all the boreholes are instantaneously lowered to p_1, and are subsequently maintained unchanged. Starting from this moment, the plane filtration of a two-phase liquid starts in the stratum, in the direction of the boreholes. Due to the symmetrical arrangement and to the maintenance of an unchanged pressure p_1, the whole stratum is divided into n identical angular sectors, each of which feeds a single borehole of the battery.

The boundary of a sector is impermeable and consists of two radii and a section of the external boundary of the stratum. A borehole located inside of a sector works as a result of the energy of the gas dissolved in the petroleum. The central angle, determining the sector, is equal to $2\pi/n$. From considerations of symmetry, it is possible to consider only one half of a sector. This half of a sector has a central angle equal to π/n. We locate the origin of coordinates at the center of the original round stratum, and the x axis is directed through the borehole. The flow of the gasified liquid inside the last sector is two-dimensional. It is described by the equations

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$$\frac{\partial p}{\partial t} = \frac{k}{m_1\mu_1}\left[\frac{\partial}{\partial x}\left(Fp\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(Fp\frac{\partial p}{\partial y}\right)\right]$$
$$\frac{\partial \rho}{\partial t} = \frac{k}{m_1\mu_2}\left[\frac{\partial}{\partial x}\left(F_2\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(F_2\frac{\partial p}{\partial y}\right)\right]$$
$$F = F_1 + \frac{\mu_1}{\mu_2}F_2 \qquad (1)$$

where μ_1, μ_2 are the viscosities of the gas and the petroleum; F_1, F_2 are the relative phase permeabilities of the gas and the petroleum.

Within the framework of a minimal sector, the initial and boundary conditions of the problem assume the form

$$\begin{aligned}
&p = p_0,\ \rho = \rho_0 \text{ with } t = 0\\
&\partial p/\partial r = 0,\ \partial\rho/\partial r = 0 \text{ with } r = 0\\
&\partial p/\partial\varphi = 0,\ \partial\rho/\partial\varphi = 0 \text{ with } \varphi = 0\ \ (r \neq r_1)\\
&\partial p/\partial l = 0,\ \partial\rho/\partial l = 0 \text{ with } \varphi = \pi/n\\
&p = p_1 \text{ with } r = r_1\ \ (\varphi = 0)\\
&\partial p/\partial r = 0,\ \partial\rho/\partial r = 0 \text{ with } r = r_2
\end{aligned} \qquad (2)$$

where φ is the angle formed by the radius vector of an arbitrary point of the stratum and the positive direction of the x axis; l is the external normal of the boundary of the sector with $\varphi = \pi/n$.

It is required to find the solution of the system of nonlinear differential equations (1) with the initial and boundary conditions (2).

We go over to polar coordinates, using the formulas

$$x = r\cos\varphi,\ y = r\sin\varphi$$

We introduce the dimensionless quantities:

$$R = \frac{r_2}{r},\ P = \frac{p}{p_0},\ \theta = \frac{n}{\pi}\varphi,\ \tau = \frac{kp_0}{m_1\mu_1 r_2^2}t \qquad (3)$$

Then, system (1) reduces to the following:

$$\frac{\partial P}{\partial\tau} = R^2FP\left\{R^2\left[\frac{\partial^2P}{\partial R^2} + \frac{1}{R}\frac{\partial P}{\partial R} + \frac{1}{P}\left(\frac{\partial P}{\partial R}\right)^2 + \frac{F'}{F}\frac{\partial P}{\partial R}\frac{\partial\rho}{\partial R}\right] + \frac{n^2}{\pi^2}\left[\frac{\partial^2 F}{\partial\theta^2} + \frac{1}{P}\left(\frac{\partial P}{\partial\theta}\right)^2 + \frac{F'}{F}\frac{\partial P}{\partial\theta}\frac{\partial\rho}{\partial\theta}\right]\right\}$$
$$\frac{\partial\rho}{\partial\tau} = \frac{\mu_1}{\mu_2}R^2F_2\left\{R^2\left[\frac{\partial^2P}{\partial R^2} + \frac{1}{R}\frac{\partial P}{\partial R} + \frac{F_2'}{F_2}\frac{\partial P}{\partial R}\frac{\partial\rho}{\partial R}\right] + \frac{n^2}{\pi^2}\left[\frac{\partial^2P}{\partial\theta^2} + \frac{F_2'}{F_2}\frac{\partial P}{\partial\theta}\frac{\partial\rho}{\partial\theta}\right]\right\} \qquad (4)$$

with the corresponding dimensionless initial and boundary conditions. This system cannot be solved analytically. Therefore, we use the method of finite-difference equations. We divide the original sector into small sector bands. Let the radius of the sector r_2 be divided into N equal parts with the spacing $h = r_2/N$. If the points of the division are numbered in reverse order, they can be calculated using the formula $r^{(i)} = r_2 - ih$ ($i = 0, 1, 2, \ldots, N$).

We note that the replacement of the variables r and φ by the variables R and θ using substitutions (3) maps the sector in which the solution of the system of equations is sought on an infinite sector band, bounded by another circle of radius $R = 1$ with a central angle $\theta = 1$ and by two radii of infinite length. As a result of this, the boundary conditions with $r = 0$ go over into the analogous conditions with $R = \infty$. This circumstance makes it difficult to take the given boundary condition into consideration. We correct this condition in the following manner. We calculate the points of division of the variable R_i using the formula

$$R_i = r_2/(r_2 - ih)$$

To avoid using the boundary condition with $R_N = \infty$, we rewrite this condition at the point R_{N-1}. Then, the infinite radius $1 \le R < \infty$ is replaced by the finite radius $1 \le R \le R_{N-1}$. Taking the center of the stratum as the center of a circle, we draw arcs of the circle with the radii $R = R_1$. As a result, they are correspondingly mapped on the dimensionless sector band. We note that the points of division of the dimensionless coordinate are arranged with a variable spacing $R_{i+1} - R_i$ ($i = 0, 1, 2, \ldots, N-1$).

We pass on to division of the region with respect to the coordinate $\varphi = \pi/n$ into M equal parts with a spacing $\Delta\varphi = \pi/nM$. The corresponding spacing $\Delta\theta = 1/M$. The points of division φ_j and θ_j ($j = 0, 1, 2, \ldots, M$) will be unequally spaced, and are calculated using the formulas

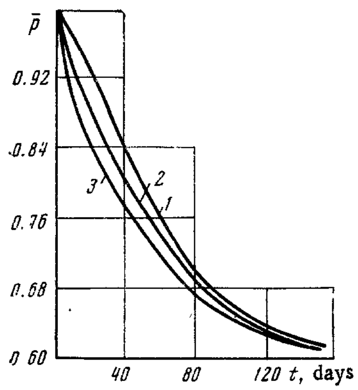

Fig. 1

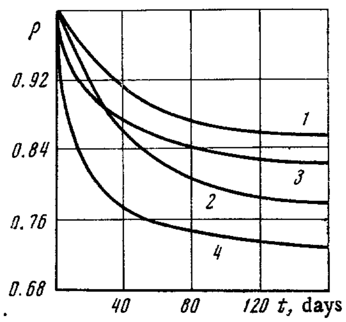

Fig. 2

$$\varphi_j = j\Delta\varphi,\ \theta_j = j\Delta\theta$$

We draw the straight lines $\varphi = \varphi_j$ (j = 0, 1, 2, ..., M). They have a corresponding mapping in the dimensionless region. As a result of taking combined account of the divisions with respect to both the coordinates φ and r, we obtain small curvilinear trapezoids. Their apexes are the mesh points of a grid region obtained by the intersection of the straight lines $\varphi = \varphi_j$ and the arcs of circles with the radii $r = r_i$. The set of all the apexes of the curvilinear trapezoids forms a grid region. The subdivision of the region in which Eqs. (4) are given with respect to the time τ is carried out starting from the spacing $\Delta\tau$. The subsequent values of the time, determining the layers, are calculated using the formula $\tau_k = k\Delta\tau$ (k = 0, 1, 2, ...). Substituting partial derivatives [1] into (4), and making simple transformations, we obtain its finite-difference analog for every internal point (τ_k, R_i, θ_j)

$$\begin{aligned}
P_{ijk+1} = {} & P_{ijk} + \Delta\tau R_i^2 F_{ijk} P_{ijk} \Bigg\{ R_i^2 \Bigg[\frac{P_{i+1jk} - P_{ijk}}{(R_i - R_{i-1})(R_{i+1} - R_i)} - \frac{P_{ijk} - P_{i-1jk}}{(R_i - R_{i-1})^2} + \\
& + \frac{F'_{ijk}}{F_{ijk}} \frac{\rho_{i+1jk} - \rho_{i-1jk}}{R_{i+1} - R_{i-1}} + \frac{1}{P_{ijk}} \frac{P_{i+1jk} - P_{i-1jk}}{R_{i+1} - R_{i-1}} + \frac{1}{R_i} \Bigg) \frac{P_{i+1jk} - P_{i-1jk}}{R_{i+1} - R_{i-1}} \Bigg] + \\
& + \frac{n^2}{\pi^2} \Bigg[\frac{P_{ij+1k} - 2P_{ijk} + P_{ij-1k}}{(\Delta\theta)^2} + \Bigg(\frac{F'_{ijk}}{F_{ijk}} \frac{\rho_{ij+1k} - \rho_{ij-1k}}{2\Delta\theta} + \frac{1}{P_{ijk}} \frac{P_{ij+1k} - P_{ij-1k}}{2\Delta\theta} \Bigg) \frac{P_{ij+1k} - P_{ij-1k}}{2\Delta\theta} \Bigg] \Bigg\} \\
\rho_{ijk+1} = {} & \rho_{ijk} + \Delta\tau \frac{\mu_1}{\mu_2} R_i^2 F_{2ijk} \Bigg\{ R_i^2 \Bigg[\frac{P_{i+1jk} - P_{ijk}}{(R_i - R_{i-1})(R_{i+1} - R_i)} - \frac{P_{ijk} - P_{i-1jk}}{(R_i - R_{i-1})^2} + \\
& + \Bigg(\frac{F'_{2ijk}}{F_{2ijk}} \frac{\rho_{i+1jk} - \rho_{i-1jk}}{R_{i+1} - R_{i-1}} + \frac{1}{R_i} \Bigg) \frac{P_{i+1jk} - P_{i-1jk}}{R_{i+1} - R_{i-1}} \Bigg] + \frac{n^2}{\pi^2} \Bigg[\frac{P_{ij+1k} - 2P_{ijk} + P_{ij-1k}}{(\Delta\theta)^2} + \frac{F'_{2ijk}}{F_{2ijk}} \frac{\rho_{ij+1k} - \rho_{ij-1k}}{2\Delta\theta} \frac{P_{ij+1k} - P_{ij-1k}}{2\Delta\theta} \Bigg] \Bigg\}
\end{aligned}$$

$$(i = 1, 2, \ldots, N-1;\ j = 1, 2, \ldots, M-1;\ k = 0, 1, 2; \ldots) \tag{5}$$

The mesh point at which the borehole is located has the index numbers (i = m, j = 0). Among relationships (5), there is none for determining the end-face petroleum saturation. We construct it starting from the values of the pressure and the petroleum saturation at the mesh points $(t_k, r_m, 0)$, $(t_k, r_{m+1}, 0)$, $(t_k, r_{m+2}, 0)$ and substituting the expressions for the partial derivatives [1] into the second equation of (4)

$$\begin{aligned}
\rho^{(1)}_{m0k+1} = {} & \rho_{m0k} + \Delta\tau \frac{\mu_1}{\mu_2} R_m^2 F_{2m0k} \Bigg\{ R_m^2 \Bigg[\frac{2}{R_{m+2} - R_m} \Bigg(\frac{P_{m+20k} - P_{m+10k}}{R_{m+2} - R_{m+1}} - \\
& - \frac{P_{m+10k} - P_{m0k}}{R_{m+1} - R_m} \Bigg) + \Bigg(\frac{F'_{2m0k}}{F_{2m0k}} \Bigg(\frac{\rho_{m+10k} - \rho_{m0k}}{R_{m+1} - R_m} - \frac{R_{m+1} - R_m}{R_{m+2} - R_m} \Bigg(\frac{\rho_{m+20k} - \rho_{m+10k}}{R_{m+2} - R_{m+1}} - \\
& - \frac{\rho_{m+10k} - \rho_{m0k}}{R_{m+1} - R_m} \Bigg) \Bigg) + \frac{1}{R_m} \Bigg) \Bigg(\frac{P_{m+10k} - P_{m0k}}{R_{m+1} - R_m} - \frac{R_{m+1} - R_m}{R_{m+2} - R_m} \Bigg(\frac{P_{m+20k} - P_{m+10k}}{R_{m+2} - R_{m+1}} - \\
& - \frac{P_{m+10k} - P_{m0k}}{R_{m+1} - R_m} \Bigg) \Bigg) \Bigg] + \frac{n^2}{\pi^2} \Bigg[\frac{P_{m2k} - 2P_{m1k} + P_{m0k}}{(\Delta\theta)^2} + \frac{F'_{2m0k}}{F_{2m0k}} \frac{(P_{m1k} - P_{m0k})(\rho_{m1k} - \rho_{m0k})}{(\Delta\theta)^2} \Bigg] \Bigg\}
\end{aligned}$$

Using the expressions for the partial derivatives of the components at the mesh point (τ_k, R_m, 0), starting from the values of the pressures and the petroleum saturation at the mesh points (t_k, $R_{m-2,0}$), (τ_K, $R_{m-1,0}$), (τ_k, $R_{m,0}$) we obtain still another value of the petroleum saturation $\rho^{(2)}_{m0k+1}$. Since the end face is subjected to an effect from both right and left, the end-face petroleum saturation can be calculated

$$\rho_{m0k+1} = (\rho^{(1)}_{m0k+1} + \rho^{(2)}_{m0k+1}) / 2 \tag{6}$$

Linearizing system (4), we can obtain the condition for the stability of the corresponding difference scheme

$$\Delta\tau \leqslant \min \left[\frac{1}{4F_{\max}} \left(\frac{h}{r_2} \right)^2, \ \frac{\rho_0}{4F_{2\max}} \left(\frac{h}{r_2} \right)^2 \right]$$

In calculations of the spacing $\Delta\tau$ and h in accordance with the scheme (5), it is recommended to connect these conditions.

The set of values of the pressure P_{ijk} and the petroleum saturation ρ_{ijk} for every moment of time τ_k characterizes the distribution of the pressure and the petroleum saturation over the stratum. The process of calculating these values is bound up with the need to carry out a large number of arithmetic and logical operations. This calculation process can only be carried out using an electronic computer.

The scheme set forth above was used to calculate an example (in a Ural-2 machine) with the following values of the starting parameters: $r_2 = 600$ m; $h = 100$ m ($N = 6$); $\Delta\tau = 0.36 \cdot 10^{-2}$ ($\Delta t = 0.5$ h); $\Delta\theta = 1/3$ ($\Delta\varphi = 6°$), or $M = 5$, $n = 6$, $\mu_1 = 0.0125$ cP, $\mu_2 = 2$ cP, $P_0 = 1$ ($p_0 = 100$ atm), $P_1 = 0.6$, $\rho_0 = 1$, $m = 3$.

The empirical functions F_1 and F_2 were taken in the form

$$F_2 = \rho^3, \ F_1 = (1 - \rho)^2$$

On the basis of the numerical results obtained, curves of the change in the pressure and the petroleum saturation as a function of the time were plotted for different points of the stratum. They are shown on Figs. 1 and 2. The point of the stratum with the coordinates $r = 300$ m and $\varphi = 0$ corresponds to the end-face of the borehole. On Fig. 1 the curve of the end-face pressure coincides with the t axis, while the curve of the end-face petroleum saturation on Fig. 2 corresponds to the coordinates $\varphi = 0$, $r = 300$ m (curve 4). It can be seen that the end-face petroleum saturation varies monotonically and has the greatest value over the whole depletion time, compared with other points of the stratum. The change in the pressure, as well as of the petroleum saturation, takes place monotonically at all points of the stratum.

Comparing curves 1, 2, 3, corresponding to the points (0 m, 0), (200 m, 0), and (500 m, 0) on Fig. 1 (for the petroleum saturation on Fig. 2), we find that, with increasing distance from the borehole along the straight line $\varphi = 0$, the pressure increases. Calculations show that, with increasing distance from the borehole along the arc of a circle with its center at the origin of coordinates, the pressure also increases. This is evidence of the fact that the change in the pressure around the borehole takes place funnel-wise; here, the funnel does not have a round cross section. The corresponding curves of the petroleum saturation around the borehole are also funnel-shaped.

At points of the external boundary of the sector, the change in the pressure and the petroleum saturation takes place with a definite lag. However, the small dimensions of the sector feeding a single borehole makes it possible for the effect to be felt rapidly by the whole sector. The point of the sector furthest from the borehold (500 m, 30°) is subjected to the effect of the borehole approximately 10 days after the battery is put into operation. The rapid effect of the borehole on the whole stratum can lead to an accelerated evolution of gas from the petroleum. To prevent this phenomenon, it is desirable to make analogous calculations for each new deposit, before undertaking the solution of the problem of establishing the value of the end-face pressure, of the number of boreholes in the battery, of determining the time of complete depletion, etc.

The results of the calculations show that, for the given example, complete depletion of a sector (or of the stratum) takes place with 160 days after the start-up of the battery.

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